

Exercise 48

Show that a solution of $x^8 - 1 = 0$ is $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

Solution

Let

$$x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i.$$

Then

$$\begin{aligned}x^8 &= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)^8 \\&= \left[\frac{\sqrt{2}}{2}(1+i) \right]^8 \\&= \left(\frac{\sqrt{2}}{2} \right)^8 (1+i)^8 \\&= \left(\frac{1}{\sqrt{2}} \right)^8 (1+i)^8 \\&= \frac{1}{16}(1+i)^8 \\&= \frac{1}{16}[(1+i)^2]^4 \\&= \frac{1}{16}(1+2i+i^2)^4 \\&= \frac{1}{16}(1+2i-1)^4 \\&= \frac{1}{16}(2i)^4 \\&= \frac{1}{16}(16i^4) \\&= i^4 \\&= 1\end{aligned}$$

Therefore, subtracting 1 from both sides,

$$x^8 - 1 = 0.$$